# THE SMALL OSCILLATIONS OF A KITE.

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## I.—INTRODUCTION.

1. A mathematical investigation of the small oscillations of a kite was proposed in "Stability in Aviation" in the form of a problem (p. 180, problem 16). A solution of this problem has been given by Prof. J. M. Bose in the Bulletin of the Calcutta Mathematical Society, Vol. II., No. 1. Unfortunately his investigation contains many serious errors of a fundamental character, the effect of which is to render the solution inapplicable to a system in any way resembling an ordinary kite. As examples, the variations in the components of the tension of the kite string are assumed to depend on the velocity components instead of on the displacements of the kite, although Prof. Bose's previous equations show the contrary to be the case. Again, while in his introduction he considers the case of a kite attached by a forked string he contradicts himself by neglecting the displacements of the point of intersection of the string with the plane of the kite. Further, in §§13, 14, he omits one of the six equations of motion which he has just written down. The paper contains other errors as well.

Mr. Berwick and I carefully examined Prof. Bose's paper, but came to the conclusion that nothing short of a complete re-investigation of the equations of motion would meet the case.

The main difficulty which distinguishes the problem of the kite from that of the aeroplane is connected with the action of the kite string. If the kite is attached by means of a forked string in the form of a Y, the three independent axes about which it can rotate without displacing the string are not all concurrent, so that even the kinematical conditions would become complicated in the case of a general solution. As it is uncertain how far it would be worth while to give such a solution the present paper is limited to the consideration of the small oscillations of the kite about its position of equilibrium. In this case the expressions for the linear and angular accelerations for fixed and moving axes become the same, and, what is more important, the conditions introduced by the forked string may be satisfied by treating the point of attachment as being different for the longitudinal and lateral oscillations. The simplest plan is then to use a different origin and axes in studying the two types of oscillation.

In this paper the equations are formulated for the general case in which the string is of finite length and extensible, and particular modifications occur when the string is inextensible and when it is practically infinite in length. Any attempt to take into account the weight and therefore also the inertia of the string would necessarily introduce the small oscillations of the string itself about its form of equilibrium in a catenary, and perhaps the wind resistance on the string would also require consideration. It might however be possible to make some more or less satisfactory assumption that would represent these effects, for example, inertia of the string might be represented by components of tension proportional to the accelerations of the point of attachment.

At the time when "Stability in Aviation" was written it seemed probable that it might be necessary to determine the stability coefficients of aeroplanes by attaching them to strings, flying them as kites and observing their periods of oscillation. The main difficulty is, of course, that the disturbances produced by fluctuations in wind velocity are calculated to render the free oscillations difficult of observation. Some preliminary mathematical work was done on the lines of the present paper by Prof. (now Lieut.) E. H. Harper, M.A., when my book was in preparation, and he obtained biquadratic equations for the oscillations. According to the present investigation the resulting equations may be of the sixth degree for longitudinal and of the fifth degree for lateral oscillations. The reason for the difference in this respect as compared with the corresponding equations for the aeroplane is, generally speaking, that, owing to the action of the string and the wind, the kite can only occupy a definite position of equilibrium, whereas the equilibrium of the aeroplane is not to the same extent dependent on its position in space.

It is convenient to start with the general case of a dynamical system possessing no special kind of symmetry, because in this case the equations of motion become symmetrical. As soon as considerations of symmetry are applied to a dynamical system, the symmetry usually disappears from its equations of motion.

I have purposely introduced the method of cross multiplication in writing down some of the expressions, because I find it of very great use in writing down the equations of motion of a rigid body or the six components of a system of forces, and it is well that anyone who is obliged to make use of these formulæ should know of this simple and convenient method of writing them down.

It has been assumed that the wind is blowing in a horizontal direction. If this be not the case, the necessary modifications in the equations should present no difficulty.

## II.—GENERAL EQUATIONS OF SMALL OSCILLATION. ORIGIN CENTRE OF MASS.

2. Take axes fixed in the body (the kite) through its centre of mass. Suppose the string is attached at a single point, and let the notation be defined as follows:—

*Tension*:—$x$, $y$, $z$, co-ordinates of point of attachment.

$S_x$, $S_y$, $S_z$, components of tension referred to directions which remain fixed *in space*, $s$ length of string.

*Gravity*:—$l$, $m$, $n$, direction cosines of gravity, *i.e.*, of the vertical.

*Air Pressure*:—$V_1$, $V_2$, $V_3$, components of the velocity with which the wind is blowing *against* the kite, or algebraically, $-U_1$, $-U_2$, $-U_3$, the components in the positive direction of the axes.

$-X$, $-Y$, $-Z$, $-L$, $-M$, $-N$, forces and couples due to air pressure as in "*Stability in Aviation*," so that $X$ represents a force in the negative direction.

*Displacements*:—Let the body receive small displacements.

$\xi$, $\eta$, $\zeta$, displacements of the centre of gravity.

$u$, $v$, $w$, corresponding velocities.

$\theta_1$, $\theta_2$, $\theta_3$, angular displacements (small) about the axes.

$p$, $q$, $r$, corresponding angular velocities.

*Inertia Coefficients*:—$W$, weight of kite (say in lbs. weight).

$A$, $B$, $C$, moments, $D$, $E$, $F$, products of inertia about axes in lbs. $\times$ ft.$^2$

3. **Changes in constants due to displacement.**—In consequence of the rotation of the axes the above vectors representing the components of tension, gravity and wind velocity will be altered in the displaced position, and the changes can be written down by the method of cross multiplication in the usual way. If $\alpha$, $\beta$, $\gamma$,

be components of any such vector, the cross multiplication for transforming from the old to the new axes stands thus:—

$$\begin{array}{lccccccc} \text{Old} & \alpha & & \beta & & \gamma & & \alpha \\ & \theta_1 & \times & \theta_2 & \times & \theta_3 & \times & \theta_1 \end{array}$$

New

$$\alpha^1 = \alpha + \beta\theta_3 - \gamma\theta_2 \qquad \beta^1 = \beta + \gamma\theta_1 - \alpha\theta_3 \qquad \gamma^1 = \gamma + \alpha\theta_2 - \beta\theta_1$$

Thus, for the new components of

*Tension* $Sx + \theta_3 Sy - \theta_2 Sz$ . . . . . . . (1)
*Gravity* $W\,(l + \theta_3 m - \theta_2 n)$ in lbs. weight . . . (2).
*Wind Velocity* $V_1 + \theta_3 V_2 - \theta_2 V_3$ . . . . . . (3)

*The variations of the air resistance* will thus contain terms in the resistance derivatives due to the angular displacement as well as to the velocity components, and they assume the form

$$X = X_o + (u + \theta_3 V_2 - \theta_2 V_3)\, X_u + \text{etc.} \ldots + rX_r \qquad (4)$$
$$N = N_o + (u + \theta_3 V_2 - \theta_2 V_3)\, N_u + \ldots + rN_r \qquad (5)$$

*For the displacement in space of the point of attachment,* the rule of cross multiplication reads the reverse way.

$$\begin{array}{ccccccc} \theta_1 & & \theta_2 & & \theta_3 & & \theta_1 \\ & \times & & \times & & \times & \\ x & & y & & z & & x \end{array}$$

$$\xi^1 = \xi - \theta_3 y + \theta_2 z \qquad \eta^1 = \eta - \theta_1 z + \theta_3 x \qquad \zeta^1 = \zeta - \theta_2 x + \theta_1 y$$

*For the moments of the tension* about the centre of gravity the rule of cross multiplication reads

$$\begin{array}{lccccc} \text{Point of application} & x & & y & & z \\ \text{Force} & S_x + \theta_3 S_y - \theta_2 S_z & \times & S_y + \theta_1 S_z - \theta_3 S_x & \times & S_z + \theta_2 S_x - \theta_1 S_y \end{array}$$

Couples

$$\left\{ \begin{array}{l} y\,(S_z + \theta_2 S_x - \theta_1 S_y) \\ -z\,(S_y + \theta_1 S_z - \theta_3 S_x) \end{array} \right\} \left\{ \begin{array}{l} z\,(S_x + \theta_3 S_y - \theta_2 S_z) \\ -x\,(S_z + \theta_2 S_x - \theta_1 S_y) \end{array} \right\} \left\{ \begin{array}{l} x\,(S_y + \theta_1 S_z - \theta_3 S_x) \\ -y\,(S_x + \theta_3 S_y - \theta_2 S_z) \end{array} \right\} \qquad (6)$$

4. *The linear and angular components of the mass accelerations* when expressed in lbs. weight are *in the case of small oscillations only.*

$$W\frac{du}{gdt} \text{ or } W\frac{d^2\xi}{gdt^2}, \quad W\frac{dv}{gdt} \text{ or } W\frac{d^2\eta}{gdt^2}, \quad W\frac{dw}{gdt} \text{ or } W\frac{d^2\zeta}{gdt^2} \qquad (7)$$

and

$$A\frac{dp}{gdt} - F\frac{dq}{gdt} - E\frac{dr}{gdt} \text{ or } A\frac{d^2\theta_1}{gdt^2} - F\frac{d^2\theta_2}{gdt^2} - E\frac{d^2\theta_3}{gdt^2} \quad . \qquad (8)$$

and similar expressions for the corresponding couples about the other axes.

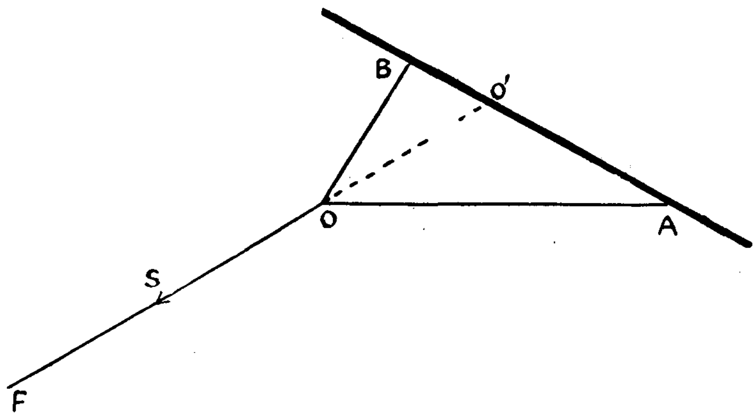
*To write down the equations of motion* it is only necessary to equate the following:—

$$\text{Linear mass acceleration of (7)} = \left\{ \begin{array}{l} \text{Tension component of (1).} \\ +\ \text{gravity component of (2).} \\ -\ \text{air resistance component of (4).} \end{array} \right. \qquad (9)$$

$$\text{Angular mass acceleration of (8)} = \left\{ \begin{array}{l} \text{Tension moment of (6).} \\ -\ \text{air resistance moment of (5).} \end{array} \right. \qquad (10)$$

and there would be no advantage in our spelling these equations out in full. It will be observed that these equations necessarily contain the angular displacements $\theta_1$, $\theta_2$, $\theta_3$, as well as the corresponding velocities $p$, $q$, $r$, and therefore they may be expected in the general case to lead to period equations of higher degree than is the case with the aeroplane.

## III.—ACTION OF THE STRING.




5. The character of the oscillations will depend very largely on the mode of attachment of the string and on the manner in which its tension varies when the kite is displaced.

CASE I.—**String weightless and inextensible.**—The simplest cases are those in which the string is assumed to be weightless and inextensible and attached to a fixed point F in the ground. In such cases the kite is perfectly free to rotate about F, but cannot move in the direction of the string.

In many kites, moreover, the attachment is forked as shown in the figure. In these the following displacements are possible:—

1. The kite and string may turn about F, its direction of motion being perpendicular to OF and either in or perpendicular to the plane of the figure. This gives two possible displacements.
2. It may turn about an axis through O perpendicular to the plane of the figure, the string remaining at rest.
3. It may similarly turn about OF.
4. It may turn about the line AB.

Let FO produced meet AB in O′. Then it will be seen that when the string OF remains at rest

O is the centre of rotation for longitudinal displacements,
O′ is the centre for lateral displacements.

[The kite is, of course, *able* to turn about any axis through O whatever, but unless the three strings OA, OB, OF, remain in one plane the tensions in them cannot be in equilibrium, and such a rotation is statically impossible unless O is weighted, which is not supposed the case.]

It follows that *the point of attachment of the string must be taken to be O for longitudinal and O′ for lateral oscillations.*

If $S$ is the tension of the string and if the point of attachment undergoes a *small* displacement $d_1$ perpendicular to the string, the length of the string up to the point of attachment being $s$, it is easily seen that the component of the tension tending to bring the kite back again is $Sd_1/s$ to the necessary order of approximation. If $s$ is extremely long this component becomes negligible and the equilibrium becomes neutral for displacements perpendicular to the string.

6. CASE II.—**String extensible.**—In this case if the kite is displaced towards F through a distance $d_2$ the tension is changed from $S_0$ to $S_0 - Ed_2/s$ where $E$ is the modulus of elasticity.

Now it will be seen that in the case of the inextensible string we have the geometrical condition $d_2 = O$, but that the tension $S$ is indeterminate in the displaced position. Thus, it would be necessary to eliminate $S$ from the equations of motion by means of this geometrical condition. It is clear that with the centre of gravity as origin the equation of moments also becomes very complicated, and therefore the work is greatly simplified by taking as origin the point of attachment, $O$ for longitudinal and $O'$ for lateral oscillations, so that the moment of the tension components about the origin vanishes.

## IV.—ORIGIN AT POINT OF ATTACHMENT.

7. In this case the expressions of § II. will be modified as follows:—

$-x, -y, -z$, will now be co-ordinates of the centre of gravity.

$\xi^1, \eta^1, \zeta^1$, displacements of point of contact.

$V_1, V_2, V_3$, will be unaltered, but *it will be necessary to refer the resistance components and resistance derivatives to the new origin. The values of these will therefore be entirely different from those of* § II. and will only be deducible from them by the formulæ of transformation (*Stability in Aviation*, § 30, and corresponding generalisations for space in three dimensions). With this assumption the forms in which the resistance derivatives occur will be the same as in § II.

The couples due to the tension will now vanish, but instead we shall have couples due to gravity which may be written down by the cross multiplication method.

| | | | |
|---|---|---|---|
| Point of application | $-x$ | $-y$ | $-z$ |
| Force | $W(l+\theta_3 m-\theta_2 n)$ | $W(m+\theta_1 n-\theta_3 l)$ | $W(n+\theta_2 l-\theta_1 m)$ |

Couples $W\left\{z(m+\theta_1 n-\theta_2 l)-y(n+\theta_2 l-\theta_1 m)\right\}$ and two similar. (11)

8. Finally the components of mass acceleration of the centre of gravity are *for small oscillations only.*

$$W\left(\frac{d^2\xi^1}{gdt^2}+\frac{y}{g}\frac{d^2\theta_3}{dt^2}-\frac{z}{g}\frac{d^2\theta_2}{dt^2}\right) \text{ or } W\left(\frac{du^1}{gdt}+\frac{y}{g}\frac{dr}{dt}-\frac{z}{g}\frac{dq}{dt}\right) \text{ etc.} \quad (12)$$

and the corresponding couples due to rotation take the form

$$A\frac{dp}{gdt}-F\frac{dq}{gdt}-E\frac{dr}{gdt}$$

$$-yM\left(\frac{dw^1}{gdt}+\frac{x}{g}\frac{dq}{dt}-\frac{y}{g}\frac{dp}{dt}\right)+zM\left(\frac{dv^1}{gdt}+\frac{z}{g}\frac{ds}{dt}-\frac{x}{g}\frac{dr}{dt}\right)$$

which reduce to

$$A^1\frac{dp}{gdt}-F^1\frac{dp}{gdt}-E^1\frac{dr}{gdt}-yM\frac{dw^1}{gdt}+zM\frac{dv^1}{gdt} \quad . \quad . \quad . \quad (13)$$

where

$$A^1 = A + M(y^2+z^2) \qquad E^1 = E + Mxz \qquad F^1 = F + Mxy$$

so that $A^1$, $B^1$, $C^1$, $D^1$, $E^1$, $F^1$ are the moments and products of inertia about the new axes.

The equations, therefore, now assume the final forms

$$W(\dot{u}+yr-z\dot{q})/g=S_x+\theta_3 S_y-\theta_2 S_z+W(l+\theta_3 m-\theta_2 n)$$
$$-X_0-(u+\theta_3 V_2-\theta_2 V_3)X_u-\ldots-rX_r \quad . \quad . \quad (14)$$
$$\{A^1\dot{p}-F^1\dot{q}-E^1\dot{r}+M(z\dot{v}-y\dot{w})\}/g=W\{z(m+\theta_1 n-\theta_3 l)-y(n+\theta_2 l-\theta_1 m)\}$$
$$-L_0-(u+\theta_3 V_2-\theta_2 V_3)L_u-\ldots-rL_r \quad . \quad . \quad (15)$$

## V.—LATERAL OSCILLATIONS.

9. In this case the origin must be taken at $O^1$ and the equations can further be simplified by *taking the axis of u horizontal*, so that we have $V_2=O=V_3$, $l=o=n$, $m=1$. Also we may put $V_1=U$ where $U$ is the actual velocity of the wind. Let $\beta$ be the inclination of the string to the horizon. Putting $S_x=S\cos\beta$ and $S_y=S\sin\beta$, the equations give, with $\zeta$, $\theta_1$, $\theta_2$, proportional to $e^{\lambda t}$

$$W\lambda^2(\zeta+x\theta_2-y\theta_1)/g=-W\theta_1-Z_w(\lambda\zeta+U\theta_1)-\lambda(\theta_1 Z_p+\theta_2 Z_q)$$
$$-S\zeta/S_p+S(\theta_2\cos\beta-\theta_1\sin\beta) \quad . \quad . \quad . \quad (16)$$
$$\lambda^2(A\theta_1-F\theta_2-\zeta My)/g=Wy\theta_1-L_w(\lambda\zeta+U\theta_1)-\lambda(\theta_1 L_p+\theta_2 L_q) \quad (17)$$
$$\lambda^2(\beta\theta_2-F\theta_1+\zeta Mx)/g=-Wx\theta_1-M_w(\lambda\zeta+U\theta_1)-\lambda(\theta_1 M_p+\theta_2 M_q)(18)$$

If we eliminate $\theta_1$ from the second and third of these equations $\lambda$ occurs as a factor of the resulting equation. Dividing out this factor we obtain an equation of the fifth degree for the period equation in $\lambda$.

10. CASE I.—*In the case of an infinitely long string* where $S\zeta/s$ is negligible we may put $\lambda\zeta=w$, and the displacement $\zeta$ does not occur in the equations by itself. In such a case the equilibrium is obviously neutral for displacements along the axis of $z$. The $\lambda$ equation again reduces to the fifth degree, but as the eliminant of the second and third equations does not in general now contain $\lambda$ as a factor, the equations for $\lambda$ cannot be again reduced in degree.

For the purpose of studying the *oscillations of the kite itself* it is, however, better to take $s=\infty$, as if it should be necessary it would always be possible to ascertain the effects of shortening the string in the form of small corrections. The *stability of position* dependent on $s$ being finite can be investigated separately by neglecting all but first powers of $\lambda$ in the period equation.

11. CASE II.—*In the case of a plane kite without keels or auxiliary surfaces*, offering no tangential resistance, if we suppose $\alpha$ to be the inclination of its plane to the horizon, it is clear that $Z=O$ and $L\sin\alpha+M\cos\alpha=O$ for all displacements, the latter equation representing the fact that no couple can be set up tending to rotate the kite in its own plane.

In this case the equations simplify. For multiplying (2) by $\sin\alpha$, (3) by $\cos\alpha$ and adding and omitting the terms which vanish in (1) we have the two equations

$$W\lambda^2(x_2-y\theta_1)/g+W(\zeta\lambda^2/g+1)\theta_1=S(\theta_2\cos\beta-\theta_1\sin\beta) \quad . \quad . \quad . \quad (19)$$
$$\lambda^2\left\{(A\theta_1-F\theta_2)\sin\alpha+(\beta\theta_2-F\theta_1)\cos\alpha\right\}/g$$
$$+W(\zeta\lambda^2/g+1)(x\cos\alpha-y\sin\alpha)\theta_1=O \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

These are evidently the equations obtained by resolving and taking moments in the plane of the kite. At the same time the equation apparently in $\lambda$ does not usually reduce to a lower degree. If, however, there is no lateral displacement of the centre of pressure when the wind blows sideways on the kite $L_w$ and $M_w$

will vanish, and the terms involving $\lambda\zeta$ will vanish. In this case $\lambda^2$ will occur as a factor of $\zeta$ throughout and dividing by this the equation reduces to a biquadratic.

CASE III.—*If the point of attachment is fixed,* $\zeta=o$ and 17, 18 lead to a biquadratic in $\lambda$.

## VI.—LONGITUDINAL OSCILLATIONS.

12. In this case the origin, in the case of a forked string, must be taken at the fork, also it is important to take the axis of $x$ along the direction of the string, and its inclination to the horizon being $\beta$ we have

$l=\sin\beta,\ m=\cos\beta,\ n=o.$
$V_1=U\cos\beta,\ V_2=U\sin\beta$ where $U$ is the horizontal wind velocity.

Also $\zeta$, $\theta_1$, $\theta_2$, and their differential coefficients vanish, and the equations become on assuming $\xi$, $\eta$ and $\theta_3$ each proportional to $e^{\lambda}$

$$W\lambda^2(\xi+y\theta_3)/g=S_x+\theta_3S_y+W(\underline{\sin\beta}+\theta_3\cos\beta)$$
$$-\underline{X_0}-(\lambda\xi-\theta_3U_0\sin\beta)X_u-(\lambda\eta-\theta_3V\cos\beta)X_0-\lambda\theta_3X_r \quad . \quad (21)$$

$$W\lambda^2(\eta-x\theta_3)/g=S_y-\theta_3S_x+W(\underline{\cos\beta}-\theta_3\sin\beta)$$
$$-\underline{V_0}-(\lambda\xi+\theta_3U\sin\theta)V_u-(\lambda\eta-\theta_3U\cos\theta)\ Y_0-\lambda\theta_3Y_r \quad . \quad . \quad (22)$$

$$\lambda^2\left\{C\theta_3+W(\xi y-\eta x)\right\}/g=W\left\{y(\underline{\sin\beta}+\theta_3\cos\beta)-x(\underline{\cos\beta}-\theta_3\sin\beta)\right\}$$
$$-\underline{N_0}-(\lambda\dot{\xi}+\theta_3U\sin\beta)N_u-(\lambda\eta-\theta_3U\cos\beta)\ N_u-\lambda\theta_3N_r \quad . \quad (23)$$

13. The conditions of equilibrium give with $S_x=S_0$ and $S_y=O$

$$S_0+W\sin\beta-X_0=O \quad . \quad . \quad . \quad (24)$$

$$W\cos\beta-\gamma_0=O \quad . \quad . \quad . \quad (25)$$

$$W(y\sin\beta-x\cos\beta)-N_0=O \quad . \quad . \quad . \quad (26)$$

in virtue of which the terms underlined vanish, the constant portion of the tension in $S$ being cancelled in consequence. Taking the most general case of an extensible string of finite length we thus obtain

$$W\lambda^2(\xi+y\theta_3)/g=-(\lambda\xi+\theta_3U\sin\beta)X_u-(\lambda\eta-U\theta_3\cos\beta)\ X_v-\lambda\theta_3X_r$$
$$-E\xi/s+W\theta_3\cos\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

$$W\lambda^2(\eta-x\theta_3)/g=-(\lambda\xi+\theta_3U\sin\beta)Y_u-(\lambda\eta-\theta_3U\cos\beta)\ Y_v-\lambda\theta_3Y_r$$
$$-S_0(\eta/s+\theta_3)-M\theta_3\sin\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

$$\lambda^2\left\{C\theta_3+W(\xi y-\eta x)\right\}/g$$
$$=-(\lambda\xi+\theta_3U\sin\beta)N_u-(\lambda\eta-\theta_3U\cos\beta)\ N_v-\lambda\theta_3N_r$$
$$W\theta_3(y\cos\beta+x\sin\beta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

In the most general cases of an extensible string of finite length these lead to a period equation of the sixth degree, but the following modifications occur.

14. CASE I.—**String finite but inextensible.**—Here $\xi=O$, and we can write $\Delta S$ for $-E\xi/s$, where $\Delta S$ is the change in tension due to the small displacements and is small. In this case we must put $\xi=O$ in the second and third equations and these lead to a biquadratic equation for $\lambda$.

15. CASE II.—**String infinitely long and inextensible.**—Here $\xi=O$ as before, but $\eta$ does not occur in the second equation without the factor $\lambda$. Replacing $\lambda\eta$ by $v$, $\lambda$ only occurs in the first degree in the terms containing $v$ and the resulting equation for $\lambda$ is a cubic.

The extraneous solution $\lambda = 0$ depends on the fact that equilibrium is neutral for changes of position of the kite in a direction perpendicular to the string.

If the string is very long the value of $\lambda$ corresponding to the last mentioned solution is very small and hence the stability of position of the kite can be investigated approximately by neglecting all powers of $\lambda$ above the first in (27), (29) and their eliminants. In this case we may regard the oscillations of the kite itself as given to a first approximation by the solution corresponding to $S = \infty$, the effects of the finite length taking the form of small corrections.

16. CASE III.—**String very extensible.**—The opposite case to the last one is that in which the string, besides being infinitely long, is very extensible, so that the term $-E\xi/s$ is very small. In this case stability of position is very slight for displacements parallel to either axes, and it can, as before, be investigated by only retaining the lowest powers of $\lambda$ in the three equations, while the oscillations of the kite proper can now be determined approximately by neglecting both $\xi/s$ and $\eta/s$ and putting $\lambda\xi = u$, $\lambda\eta = v$. All three equations have to be used, but they involve linear functions of $u$ and $v$ and quadratic functions of $\theta$, hence the eliminant is of the fourth degree in $X$. It is to be observed that in the second equation the term $S_0\theta_3$ is *not* to be neglected under any circumstances.

17. CASE IV.—**Point of attachment fixed.**—In this case $\xi = 0$, $\eta = 0$; equations (27) and (28) determine the tension components at any instant and (29) becomes a quadratic equation for the values of $\lambda$, the oscillations depending only on the co-ordinate $\theta_3$.

There should now be no difficulty in discussing any further applications that may be of interest, such as the conditions of stability. There are, of course, other methods of forming the equations of small oscillation, and it will conduce to accuracy in such cases if the formulæ so obtained are compared and checked with the present ones. It will, however, be seen that Prof. Bose's formulæ do not agree with those of the present investigation, and that his period equations are not of the right degree.

Mr. W. E. H. Berwick has kindly checked the formulæ in this paper, and it is hoped that they are correct; failing this, that any unintentional slips may be discovered and corrected without difficulty.